Zernike analysis of all-sky night brightness maps

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An increasing number of amateur and professional-grade cameras are taking all-sky images containing spatially resolved information of the night sky brightness.

Acquiring these images on a regular basis will provide a valuable database to analyze the time evolution of the night sky brightness at local and global levels, as well as to validate theoretical models of light pollution propagation.
Three ASTMON™ all-sky maps of the night sky brightness taken from UCM Observatory at Madrid, Sept 14th, 2010, 02:04 (UTC), in the B, V and R bands. Scale in mag/arcsec²
Each all-sky image (or map) contains a huge amount of information and has several MB size.

For long-term, large-scale or time-resolved all-sky monitoring projects this poses two challenges:
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For long-term, large-scale or time-resolved all-sky monitoring projects this poses two challenges:

How to handle, store and transmit such a big amount of data?

How to make sense of them, extracting useful knowledge without getting lost in the details?
Can we reduce the amount of data needed to represent accurately the all-sky brightness maps without losing accuracy?

Can we filter out unnecessary information e.g. celestial pointlike sources like bright stars and planets?

Can we obtain some additional knowledge after carrying out this process?
There is an efficient way frequently used in science to address this kind of problems:
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Expand the complex all-sky map as a weighted sum of simpler ones.
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Expand the complex all-sky map as a weighted sum of simpler ones:

\[ \text{Complex All-Sky Map} = a \text{ (Simple Map)} + b \text{ (Simple Map)} + c \text{ (Simple Map)} + d \text{ (Simple Map)} + e \text{ (Simple Map)} + f \text{ (Simple Map)} + \ldots \]
If we know the coefficients of this expansion we know the original map.

\[ = \{ a, b, c, d, e, f, \ldots, M (\infty?) \} \]

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WISH LIST:
\[ = \{ a, b, c, d, e, f, ..., M (\infty?) \} \]

WISH LIST:
- The reconstructed map shall be equal to the original one within the precision of our measurements
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- Each coefficient (alone or in combination with others) shall give us useful information about the structure of the sky brightness.

\[ \{ a, b, c, d, e, f, ..., M (\infty?) \} \]
Does such set of basic maps exist...?
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The answer is

YES:
THERE ARE SEVERAL POSSIBLE CHOICES

We have explored the performance of the

ZERNIKE CIRCLE POLYNOMIALS
Frits Zernike (1888 – 1966) “Dutch physicist and winner of the Nobel prize for physics in 1953 for his invention of the phase contrast microscope, an instrument that permits the study of internal cell structure without the need to stain and thus kill the cells.”
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\[
Z_{n, m}(r) = N_{n, m} R_{n, m}(r) A_m(\theta),
\]

\[
N_{n, m} = \sqrt{2 - \delta_{m0}} (n+1)^{\frac{1}{2}},
\]

\[
A_m(\theta) = \begin{cases} 
\cos m\theta, & \text{for } m \geq 0 \\
\sin |m|\phi, & \text{for } m < 0 
\end{cases}
\]

\[
R_{n, m}(r) = \sum_{l=0}^{(n-m)/2} \frac{(-1)^l (n-l)!}{l! [0.5(n+m) - l]! [0.5(n-m) - l]!} r^{n-2l}
\]

\[
\frac{1}{\pi} \int_{\theta=0}^{2\pi} \int_{r=0}^{1} Z_{n, m}(r) Z_{n', m}(r) r \, dr \, d\theta = \delta_{nm} \delta_{mm'}
\]
What additional knowledge can we get...?

- The first coefficient gives us the **average sky brightness**.

- The sum of the squares of the remaining ones gives us the **variance of the brightness distribution** across the hemispherical field.

- Coefficients corresponding to Zernike modes with \( m = 0 \) inform us about the **rotationally symmetric** contributions to the overall map.

- ...
What additional knowledge can we get...?

Time course of the average sky brightness

Time course of the standard deviation of the sky brightness
The previous results correspond to maps in $\text{mag/arcsec}^2$

Maps in $\text{mag/arcsec}^2$ are quite interesting and useful, but in these units the contributions of different sources to the overall brightness are not additive

To fully exploit additivity we shall use weighted radiance maps ("cd"/m$^2$)

How do behave the Zernike fits?
R band results for weighted radiance maps (data in "cd"/m²)
More complex all-sky images could also be described that way...

... increasing the number of Zernike terms included in the expansion
Conclusions

Decomposing all-sky maps into Zernike polynomials is a very efficient way for compressing sky brightness data without losing relevant information.

The Zernike coefficients provide easy and direct access to relevant features of the sky brightness structure (average, rms deviation, anisotropy...)

The reconstructed maps strongly attenuate the effects of noise and even allow for moderate extrapolations behind obstacles close to the horizon.

Applied with user-friendly tools and a consistent protocol, the Zernike expansions may enable the realization of large-scale projects of all-sky monitoring.
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