

# Addendum to “Does Lipschitz with Respect to $x$ Imply Uniqueness for the Differential Equation $y' = f(x, y)$ ?” by Cid and López Pouso

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The main result in [1] is Theorem 2.1. Its most striking consequence is [1, Theorem 3.1], a uniqueness result for first-order ordinary differential equations with nonlinearities which satisfy a Lipschitz condition with respect to the independent variable. An earlier specific proof of [1, Theorem 3.1] was given in [2] in 1999, as we already acknowledged in the concluding remarks of [1].

We have recently found out that [1, Theorem 3.1] is a particular case to an even older theorem stated and proved by Stettner and Nowak in 1989 (see [3] and take  $d_x = 0$  and  $d_t = 1$  in the main result). Stettner and Nowak’s paper is written in German, a language which, unfortunately, we do not know, and probably we simply overlooked it when consulting bibliographic databases. Anyway, we were not aware of the result in [3] until Cid attended Nowak’s lecture at The 8th AIMS Conference on Dynamical Systems, Differential Equations and Applications (Dresden, Germany, May 25–28, 2010).

Finally, it is worthy of remark that uniqueness results deduced by means of [1, Theorem 2.1] have, in general, nothing to do with Lipschitz conditions, e.g. [1, Theorem 4.1], thus going beyond the scopes of the results in [2, 3].

## References

- [1] J. Á. Cid and R. L. Pouso, Does Lipschitz with Respect to  $x$  Imply Uniqueness for the Differential Equation  $y' = f(x, y)$ ?, this MONTHLY **116** (2009) 61–66.
- [2] C. Mortici, On the solvability of the Cauchy problem, *Nieuw Arch. Wiskd. IV. Ser.* **17** (1999) 21–23.
- [3] H. Stettner and C. Nowak, Eine verallgemeinerte Lipschitzbedingung als Eindeigkeitskriterium bei gewöhnlichen Differentialgleichungen. (German) [A generalized Lipschitz condition as criterion of uniqueness in ordinary differential equations] *Math. Nachr.* **141** (1989), 33–35.