Thermal fluctuations near a phase transition probed through the electrical resistivity of high-temperature superconductors

Noelia Cotón, Francisco J. Guzmán,* Manuel V. Ramallo, Alexandre Ríos,* Carolina Torrán, Félix Vidal
Laboratorio de Baixas Temperaturas e Superconductividade,
Departamento de Física da Materia Condensada,
Facultade de Física, Universidade de Santiago de Compostela,
Santiago de Compostela E13782 Spain.
http://lbts.usc.es

We present a simple and affordable experimental setup able to demonstrate the thermal fluctuations near a superconducting transition. By using equipment common in undergraduate laboratories, the in-plane dc electrical resistivity of a cuprate superconductor as a function of temperature is measured with resolution sufficient to analyze the fluctuation conductivity above the superconducting critical temperature, including the values of the critical exponents. We also present a simple calculation of the fluctuation conductivity within the Gaussian-Ginzburg-Landau approach, including its dependence on the layered chemical structure of the material.

I. INTRODUCTION

The fluctuation effects associated with the proximity to a phase transition are unusually large in the case of the superconducting transition of the cuprate high-temperature superconductors (HTSC), and span over an unusually wide region of temperatures.1–3 This is due mainly to their short coherence lengths and layered structure, which lead to a small and quasi-bidimensional coherent volume.1–3 In addition, they have a relatively high transition temperature $T_c$ and, therefore, the thermal agitation energy around the transition, proportional to $k_BT_c$ (where $k_B$ is Boltzmann’s constant), is large. All of this, together with the now extended commercial availability of good quality samples, opens the opportunity for simple experimental setups demonstrating not only the existence of a phase transition (as common at present in many critical phenomena undergraduate laboratories) but also measuring the effects of thermal fluctuations around that transition. In this paper, we present and discuss the measurement in HTSC of the dc electrical resistivity, $\rho(T)$, around $T_c$, using modest equipment (with a total cost of about USD 1000). The proposed experimental setup achieves the resolution necessary to perform meaningful comparisons with existing theoretical predictions on the superconducting fluctuations above $T_c$.1–6 We will also show in this paper a simple derivation, adequate for teaching, of these theory results using an approach based on the Gaussian-Ginzburg-Landau (GGL) framework. This experimental setup is already in use in undergraduate courses in our university. In fact, the experimental data here presented were obtained by students of those courses (FJG and AR).

The proposed experience is also adequate to demonstrate a system presenting reduced dimensionality, a topic of increasing relevance in modern curricula. HTSC are layered superconductors whose fluctuations may exhibit 2D, 3D, or intermediate behaviour (dimensional crossover) as the reduced-temperature $\varepsilon = (T - T_c)/T_c$ varies. This in turn is caused by the competition of the interlayer coherence length $\xi_c(\varepsilon)$ and the average interlayer distances. For instance, in YBa$_2$Cu$_3$O$_{7-\delta}$ (YBCO) both will become equal at $\varepsilon \sim 0.03$ and this compound will exhibit in our measurements dimensional crossover behaviour.

The organization of this paper is as follows: In Sect. II we describe the experimental arrangement and present measurements obtained by using it. In Sect. III we show how to extract from these data the critical temperature $T_c$ and the in-plane electrical conductivity induced by superconducting fluctuations, $\Delta \sigma_{ab}(\varepsilon)$, at temperatures $T > T_c$. In Sect. IV we present an easy derivation of the GGL equations necessary to understand such $\Delta \sigma_{ab}(\varepsilon)$ results. In Sect. V, we compare these equations with our measurements. Finally, in Sect. VI we summarize our results and some possible variations of the proposed program, in particular to adapt it to shorter or longer courses.

II. EXPERIMENTAL SETUP

We schematize in Fig. 1 the proposed experimental setup. This comprises a simple cryogenic setup rather similar to the one already presented by León-Rossano7 to measure $T_c$ in HTSC (but not the effects of thermal fluctuations), and two electrical circuits to measure in a standard four-contact configuration the resistivities of, respectively, the HTSC and a Pt-100 thermometer. If the HTSC sample is a bulk polycrystal (see below), to discriminate the thermal fluctuations a resolution of about 10$\mu$V is needed, hence a $5^{1/2}$-digit voltmeter or better is required for the HTSC circuit. Computer flat cables are appropriate for the wires running outside the sample.

*The main contribution to this work of these authors was the data taking leading to Figs. 2, 3 and 5 during their stay as undergraduate students in the Critical Phenomena Student Laboratory.
suitability. For example, bulk polycrystals of YBCO are commercially available. Optimally doped HTSC (i.e., with dopings corresponding to their maximum $T_c$, what for YBCO translates to oxygen stoichiometry $O_{\gamma-\delta}$ of $\delta \simeq 0.05$) are preferred because they are less affected by $T_c$ inhomogeneities, which would be a cause of concern (as these inhomogeneities could produce roundings around $T_c$ unrelated to thermal fluctuations). We have used in our own measurements two types of YBCO samples: bulk polycrystals of YBCO, with nearly optimal doping and obtained as detailed, e.g., in Ref. [8] (samples Y1 and Y2), and a thin film of YBCO with a thickness of $120 \pm 20$ nm and also nearly optimally doped, obtained as detailed in Ref. [9] (sample Y3). Yearly conservation of YBCO samples may be improved by preventing oxygen loss, e.g., storing them in a dry atmosphere.

In Fig. 2 we present actual $\rho(T)$ curves obtained by students (FJG and AR) in our undergraduate laboratories. Data taking was manual, not automated, and this shows up as one of the main sources of noise in these curves. The cryogenic setup cools from room temperature to near 80 K in about 15 minutes, and evolves back to room temperature in about 2 hours during which the data are taken, with special concentration in the temperatures between 0.95$T_c \leq T \leq 1.2T_c$. We note that simply replacing the outer pyrex vessel with polyurethane foam provides times appropriate for automated overnight data collection.

III. EXTRACTION OF THE FLUCTUATION-INDUCED IN-PLANE ELECTRICAL CONDUCTIVITY

The influence of the superconducting fluctuations on the resistivity will be studied by obtaining from the above $\rho(T)$ measurements the $\Delta \sigma_{ab}(\varepsilon)$ curves appropriate for comparison with the theoretical predictions. Here $\Delta \sigma_{ab}$ and $\varepsilon$ are the in-plane fluctuation-induced electrical conductivity and the reduced-temperature, whose obtainment we describe in the following subsections, III A to III D.

A. Critical temperature determination

Firstly the critical temperature $T_c$ must be obtained in order to calculate the so-called reduced-temperature $\varepsilon \equiv (T - T_c)/T_c$, which provides a measure of the proximity to the transition. As can be seen in Fig. 3(a), the resistivity transition spans over a range of temperatures as wide as 2 K. We therefore need a criterion for extracting $T_c$ from that rounded transition. We emphasize that such a criterion should lead to some temper-
ture approximately in the middle of the transition, as there are reasons for it to be rounded both above and below \( T_c \): Firstly, as it is well known, the superconducting observables will be affected by intrinsic effects such as the thermal fluctuations themselves which round the transition at both sides of \( T_c \).\(^1,12\) But also the samples may be affected by extrinsic effects. This is the case of those associated with the polycrystallinity: Focusing on the resistivity, to get zero resistance not only each superconducting grain must be superconducting but also the contact resistance between grains must be overcome by the Josephson tunneling. And the temperature for this last effect is somewhat lower than the bulk \( T_c \) of the grains.\(^1,10,11\) Therefore, we would significantly underestimate the critical temperature of the grains if we were going to use the temperature at which \( \rho \) becomes zero (or comparable to the instrumental noise) as \( T_c \).

Our criterion for determining \( T_c \) will be based on the fact that the variation of the resistivity with temperature can be expected to be maximal at the transition temperature. Therefore, we obtain \( T_c \) from the condition

\[
\frac{d\rho(T)}{dT} = \text{max at } T = T_c. \tag{1}
\]

Equivalently, this condition signals \( T_c \) as the inflexion point of the \( \rho(T) \) curve at the transition. We note here that this criterion is also supported by combined measurements of the electrical resistivity and magnetic susceptibility in YBCO.\(^12\)

To numerically compute Eq. (1), a simple and effective method is to fit the data around the transition to a third-order polynomial, and then analytically determine the inflexion point. This procedure is illustrated in Fig. 3(a) for sample Y1. The resulting \( T_c \) values are 89.3 K for sample Y1, 89.7 K for Y2, and 88.6 K for Y3. We also indicate in that figure \( \Delta T_c \), the half-width at half-maximum of the \( d\rho/dT \) peak, which provides an estimate of the uncertainty in our determination of \( T_c \) (see also sect. III D).

Let us mention here briefly an alternative criterion for \( T_c \), handy if the measurements around the transition are not clean enough so as to obtain the derivative in Eq. (1). This is \( T_{1/2} \), the temperature at which the measured \( \rho \) is half the value extrapolated from a \( \rho(T) \) linear fit at high temperatures (for instance, at 150K \( \leq T \leq 225K \); we note that performing this fit will be necessary anyway in the subsequent steps of the analysis). The value of \( T_{1/2} \) in our samples is 89.7 K for Y1, 89.7 K for Y2, and 88.9 K for Y3.

![FIG. 2: As-measured \( \rho(T) \) in the three samples studied in this work: the bulk polycrystals Y1 (squares) and Y2 (triangles) and the thin film Y3 (circles). For better clarity, only a fraction of the data set obtained for each sample is shown in this figure. The approximate sizes between voltage contacts of each samples are also indicated (illustrations are not to scale).](image)

![FIG. 3: (a) Scoop near the transition of the as-measured data for sample Y1. The solid line is the empirical fit used to determine \( T_c \) according to the criterion of \( d\rho/dT \) being maximal at \( T = T_c \) (see main text). This fit corresponds to \( \rho(T) = \rho_0 + \rho_1T + \rho_2T^2 + \rho_3T^3 \) with the constants \( \rho_0, \rho_1, \rho_2, \rho_3 \) as free parameters, the fitting region being equal to the range on which the solid line is plotted in the figure. The dotted line is \( d\rho/dT \) resulting from the fit, in arbitrary units. Its half-width at half-maximum provides an estimate to the uncertainty in the \( T_c \) determination, \( \Delta T_c \). (b) In-plane resistivity curve for sample Y1, obtained after the polycrystallinity correction indicated in the main text. The solid line corresponds to the normal-state background, obtained also as detailed in the main text. The main effect of the superconducting fluctuations is to round the experimental \( \rho_{ab} \) with respect to this background, as indicated by the shaded area in the inset. We indicate also in the inset the approximate boundaries of the regions corresponding to the different regimes of the superconducting fluctuations. The regime to be studied in this work is the conventional-GGL one, given by \( 0.02 \leq \varepsilon = (T - T_c)/T_c \leq 0.1 \). For better clarity, only a fraction of the available data is shown in these figures.](image)
B. Polycrystallinity effects

The next step is to convert the as-measured $\rho(T)$ into in-plane resistivity, $\rho_{ab}(T)$. The difference among them is as follows: A single-crystal of HTSC is strongly anisotropic, due to its layered chemical structure, formed by parallel CuO$_2$ planes with average interdistance $d$ of about 6 Å (5.85 Å in YBCO). The superconducting state itself occurs in those CuO$_2$ planes. In the normal state, electrical transport is much easier in the in-plane (or $ab$) direction than perpendicular to it. A polycrystalline HTSC, such as our samples Y1 and Y2, consists of numerous randomly-oriented single-crystal grains, each of them with the above layered structure. It will also contain structural defects which will act as essentially nonconducting grains. These features are illustrated in Fig. 4. To convert the $\rho(T)$ measurements done in polycrystals to in-plane resistivity $\rho_{ab}(T)$ curves, the polycrystallinity effects in the normal state may be modelled as $^{10,11,13}$

$$\rho(T) = \frac{1}{\alpha} (\rho_{ab}(T) + \rho_{wl}), \quad (2)$$

where $\alpha$ is a constant accounting for the fact that current paths are meandering and therefore their effective section and length are different from the sample’s dimensions (see Fig. 4), and $\rho_{wl}$ is the average contact resistivity associated with weak links among adjacent monocrystals, also constant with temperature. In the case of optimally-doped YBCO, it is known from precise measurements in high-quality single-crystals and films (see, e.g., Refs. [12, 14,15]) that the in-plane resistivity at temperatures well above the superconducting transition (for typically $T \gtrsim 2T_c$) depends linearly on the temperature and, therefore, may be written as:

$$\rho_{abB}(T) = \rho'_{abB} T, \quad (3)$$

where $\rho_{abB}(T)$ is the so-called in-plane background resistivity, and $\rho'_{abB}$ its constant temperature derivative. In optimally-doped YBCO,$^{12,14,15}$ $\rho'_{abB} = 7.5 \times 10^{-9}\Omega\text{mK}^{-1}$. There is an uncertainty in this value of about $\pm 2.5 \times 10^{-9}\Omega\text{mK}^{-1}$, due to the variations among different samples.$^{12,14,15}$ This will be accounted for in subsection III D.

From the considerations indicated above, we may easily obtain for each of the polycrystalline samples the parameters arising in Eq. (2) by just fitting to the corresponding resistivity data $\rho(T)$ in the region $2T_c \lesssim T \lesssim 2.5T_c$ a linear function in $T$, $\rho_{ab}(0) + \rho'_{ab}T$. We then obtain from those fits the $\alpha$ and $\rho_{wl}$ values as

$$\alpha = \frac{\rho'_{abB}}{\rho'_{B}}, \quad (4)$$

and

$$\rho_{wl} = \alpha \rho_B(0). \quad (5)$$

Using those $\alpha$ and $\rho_{wl}$, we apply Eq. (2) as $\rho_{ab}(T) = \alpha \rho(T) - \rho_{wl}$ to find the in-plane resistivity at all temperatures, including those near $T_c$ (see also note [13]). We show in Fig. 3(b) an example of the results obtained with this procedure.

We note here that a hypothetical mistake in the determination of the samples’ dimensions would be absorbed by the value of $\alpha$; this is why knowing the distance among voltage contacts, and keeping them parallel to each other, is not essential.

In the case of sufficiently good YBCO films, such as our sample Y3, in principle it could be considered optional to apply the polycrystallinity correction, as their room-temperature resistivity will already lead to a $\rho'_{B}$ in agreement with the single-crystal value $\rho'_{abB}$ within the 30% uncertainty of the latter. However, we stress already at this point that, even for good films, the existence of this uncertainty will be important to correctly assess the confidence of the results obtained (see also sect. III D).

A more complete account of the relationships between the electrical properties of the granular superconductors and their polycrystalline parameters $\alpha$ and $\rho_{wl}$ may be found in, e.g., Ref. [11]. It is also possible to study the polycrystallinity effects on the context of the more rigorous effective medium approach, as may be found, e.g., in Refs. [1,11,16].

C. Non-superconducting background subtraction

Finally, we must isolate from our $\rho_{ab}(T)$ curves the rounding of the transition due to thermal fluctuations near $T_c$. To do this, it is customary to introduce the so-called in-plane paraconductivity (or in-plane fluctuation conductivity), $\Delta \sigma_{ab}(\varepsilon)$, defined as$^{5,6,12,14,15}$

$$\Delta \sigma_{ab}(\varepsilon) \equiv \frac{1}{\rho_{ab}(T)} - \frac{1}{\rho_{abB}(T)}, \quad (6)$$

where the background in-plane resistivity $\rho_{abB}(T)$ corresponds to the one that would exhibit the material in
the absence of superconductivity. This $\rho_{abB}(T)$ may be obtained by performing a fit to the polycrystallinity-corrected resistivity data, $\rho_{ab}(T)$, well above the transition where superconducting effects are expected to be negligible, and extrapolating the results to lower temperatures. In the case of the optimally-doped HTSC, a simple linear fit is sufficient. Naturally, if the background fitting region coincides with the one used before to obtain $\alpha$ and $\rho_{ab}$ (see subsection III B), by construction we will obtain $\rho_{abB}(T) \approx (7.5 \times 10^{-3} \Omega \text{mK}^{-1}) T$. In Figure 3(b) we show an example of background subtraction, where we have used a background fitting region of $150 \text{K} \leq T \leq 225 \text{K}$. It is most important to always choose a background fitting region whose lower boundary does not enter in the temperature range where the fluctuation effects are analyzed already significant. In this regard, let us note that an argument based on the uncertainty principle provides the maximum temperature at which superconducting fluctuations may occur, as follows: By taking into account that the mean-field $\varepsilon$-dependence of the superconducting coherence length is $\xi(\varepsilon) = \xi(0)\varepsilon^{-1/2}$, and that the Ginzburg-Landau amplitude $\xi(0)$ relates to the actual coherence length at zero temperature as $\xi(0) \approx 0.8 \xi(T = 0 \text{K})$, both for clean and dirty superconductors, it follows that the coherence length $\xi_{c}$ above $T_{c}$ reaches its $0 \text{K}$ value at $\varepsilon \approx 0.6$. Above that reduced-temperature, any hypothetic superconducting fluctuation would have to remain confined to a volume lower than $\xi(T = 0 \text{K})$. But the latter is a limit imposed by the uncertainty principle to the shrinkage of the superconducting wave function (see, e.g., Refs. [15,17]). We therefore arrive to the conclusion that superconducting fluctuations are forbidden above $\varepsilon \approx 0.6$, or in other words they would cost a prohibitive amount of energy at those temperatures. In fact, already at $\varepsilon = 0.3$ it is $\xi(\varepsilon) \approx 1.5 \xi(T = 0 \text{K})$, so that already at $T \geq 1.3 T_{c}$ ($\approx 115 \text{K}$ in our case) we expect the uncertainty principle limitations making $\Delta \sigma_{ab}$ quite small (as in fact will be confirmed by our experimental results, see below).

We show in Fig. 5 the $\Delta \sigma_{ab}(\varepsilon)$ curves obtained by using the procedure described above, in the three samples measured. These curves also indicate the experimental uncertainty bars, and the comparison with the theoretical predictions resulting from the GGL theory. These two aspects will be discussed in what remains of the paper.

D. Main sources of experimental uncertainty

The main sources of uncertainty in the $\Delta \sigma_{ab}(\varepsilon)$ curves presented in Fig. 5 come from the background subtraction and the polycrystallinity correction (vertical error bars in that figure), and the $T_{c}$ indetermination (horizontal error bars). Note that, overall, these error bars are well larger than, e.g., the noise due to the non-automated data taking. To estimate the uncertainties of the polycrystallinity correction and background subtraction, we have varied both the reference slope $\rho'_{abB}$ used to obtain $\rho_{ab}$, and the background $\rho_{abB}$ fitting region. Specifically, the upper $\Delta \sigma_{ab}$ estimate corresponds to choosing a background fitting region of $200 \text{K} \leq T \leq 275 \text{K}$ and $\rho'_{abB} \approx 5 \times 10^{-9} \Omega \text{mK}^{-1}$, while the lower $\Delta \sigma_{ab}$ estimate corresponds to choosing a background fitting region of $125 \text{K} \leq T \leq 200 \text{K}$ and $\rho'_{abB} \approx 10^{-8} \Omega \text{mK}^{-1}$. We note that the uncertainty in $\Delta \sigma_{ab}$ associated with varying only the reference slope for the polycrystallinity correction is essentially temperature-independent and of about $\approx 30\%$ of $\Delta \sigma_{ab}$. In contrast, the uncertainty in $\Delta \sigma_{ab}$ associated with the background fitting region increases strongly with $\varepsilon$, and at approximately $\varepsilon \gtrsim 0.1$ becomes dominant over the one associated with $\rho'_{abB}$. We note also that for $\varepsilon \lesssim 0.2$ both of these uncertainties mainly
affect the amplitude of the $\Delta \sigma_{ab}(\varepsilon)$ curves in the log-log representation of Fig. 5, while the slope is affected to a much smaller extent. As will be seen in sections IV and V, it is this slope that will correspond to the critical exponent of $\Delta \sigma_{ab}(\varepsilon)$.

We also indicate in Fig. 5 the uncertainty intervals associated with the indeterminations in $T_c$, as horizontal error bars. These are obtained by varying the critical temperature to be $T_c \pm \Delta T_c$, being $\Delta T_c$ the half-width at half-maximum of the $d\rho/dT$ peaks around the transition [see Fig. 3(a)]. It may easily be seen that the $T_c$-related uncertainty becomes large below $\varepsilon \lesssim 0.02$ (roughly corresponding to $\varepsilon \lesssim 2\Delta T_c/T_c$), but for larger $\varepsilon$ it does not significantly affect the amplitude nor the critical exponents. Such an $\varepsilon$-region is marked as the “fitting region” in Fig. 5.

IV. SIMPLE CALCULATION OF $\Delta \sigma_{ab}(\varepsilon)$ IN THE GGL APPROACH IN 3D, 2D AND LAYERED SUPERCONDUCTORS

A simple calculation of the fluctuation conductivity above $T_c$ in the Ginzburg-Landau (GL) model with Gaussian fluctuations (GGL approach) for isotropic 3D-bulks and 2D-films may be found, e.g., in the textbooks Refs. [1,2]. Here we present an even somewhat simpler calculation to the free-energy density in the final expressions see, e.g., Refs. [1,2]. Here we present an even somewhat simpler calculation to the free-energy density in the final expressions see, e.g., Refs. [5,6].

We now extend the above calculation to the case of layered superconductors as is appropriate for HTSC (for a rigorous GGL treatment producing the same final expressions see, e.g., Refs. [5,6]).

We start with the usual expression of the superconducting contribution to the free-energy density in the GGL approximation above $T_c$ for isotropic 3D-bulks or 2D-films:1,2

$$
\Delta f[\Psi] = a_0 \left( \varepsilon |\Psi(r)|^2 + \xi^2(0)|\nabla \Psi(r)|^2 \right),
$$

(7)

where $\Psi$ is the superconducting wave function (whose modulus squared may be identified with a density of Cooper pairs), $a_0$ is a proportionality constant with units of energy, and $\xi(\varepsilon) \equiv \xi(0)\varepsilon^{-1/2}$ is the superconducting coherence length. In the case of 2D-films spreading over the $xy$ (or $ab$) plane, $\xi(0)$ and the gradient must be understood as their values for the in-plane direction [commonly noted also as $\xi_{ab}(0)$ and $\nabla_{xy} \Psi$]. As is well known,1,2 spatial integration of Eq. (7) leads to the equivalent expression

$$
\Delta f[\Psi] = a_0 \Psi^*(r) \left( \varepsilon \Psi(r) - \xi^2(0)\nabla^2 \Psi(r) \right),
$$

(8)

that if minimized with respect to $\Psi$ (as $\partial \Delta f/\partial \Psi^* = 0$) becomes Schrödinger-like with an effective mass $m^* = \hbar^2/[2a_0 \xi^2(0)]$.

To calculate from these formulas the energy cost of creating Cooper pairs above $T_c$, we introduce the easy step of considering a plane wave $\Psi_k(r) = \Psi_{0k} \exp(ikr)$ and substituting it into Eq. (8). This leads to

$$
\Delta f[\Psi_k] = a_0 \varepsilon_k |\Psi_{0k}|^2 \quad \text{with} \quad \varepsilon_k \equiv \varepsilon + \xi^2(0)k^2,
$$

(9)

i.e., to an energy $a_0 \varepsilon_k$ per Cooper pair. This allows us to easily calculate the conductivity in the $ab$ direction caused by those Cooper pairs by applying the Drude formula:

$$
\Delta \sigma_{ab}(\varepsilon) = \sum_k \frac{(2\varepsilon)^2 \varepsilon_k \tau_k}{m^*} \approx \frac{\pi e^2 \xi^2(0)}{4V\hbar} \sum_k \varepsilon_k^{-2}.
$$

(10)

For the last equality in the above equation we have used that $n_k$, the density of Cooper pairs produced by a fluctuating mode with wave vector $k$, must correspond to the ratio between the available thermal energy and the energy cost of such a fluctuation, i.e., $n_k = k_B T_c/(a_0 \varepsilon_k V)$, where $V$ is the sample’s volume. We have also employed that the typical lifetime $\tau_k$ of these fluctuation modes directly follows from the time version of the uncertainty principle as $\tau_k = \hbar/(k_B T_c)$. We used the result $\tau_0 = \pi \hbar/(32k_B T_c)$ that may be obtained on the grounds of microscopic theories.1,2 To perform the remaining $k$ summation, the corresponding $\Sigma$-symbol is substituted in the 3D case by $(V/8\pi^3) \int_{-\infty}^{\infty} dk_x dk_y dk_z$ and in the 2D case by $(V/4\pi^2) \int_{-\infty}^{\infty} dk_x dk_y$ (if $d$ is the film’s thickness). These integrations directly result in:

$$
\Delta \sigma_{3D}(\varepsilon) = \frac{e^2}{32h\xi(0)} \varepsilon^{-1/2}, \quad \Delta \sigma_{2D}(\varepsilon) = \frac{e^2}{16\hbar d} \varepsilon^{-1}.
$$

(11)

Note that both the critical exponents and the amplitudes vary with dimensionality.

We now extend the above calculation to the case of layered superconductors, i.e., those composed of parallel $ab$ planes separated by interplane distance $d$, the case best adapted to HTSC. As discussed, e.g., in Ref. [6], HTSC are multilayered superconductors that regarding their fluctuation effects above $T_c$ may be considered quite approximately as layered superconductors with a single interlayer distance equal to the average CuO$_2$ plane separation (e.g., 5.85 Å for YBCO). The GGL free energy functional of this type of layered superconductors is the summation of the $\Delta f[\Psi]$ functional of each $n^\text{th}$ superconducting plane plus a finite-difference term coupling adjacent planes.1,5,6

$$
\Delta f[\Psi] = a_0 \Psi_n^* (\varepsilon \Psi_n - \xi^2(0)\nabla_{xy} \Psi_n) + \frac{a_0 \xi^2(0)}{d} \left| \frac{\Psi_n - \Psi_{n+1}}{d} \right|^2.
$$

(12)

Here $\Psi_n = \Psi(x,y,nd)$ is the superconducting wave function of the $n^\text{th}$ plane, and $\xi_n(\varepsilon) = \xi(0)\varepsilon^{-1/2}$ is
the GL coherence length in the perpendicular direction. To calculate the in-plane paraconductivity that results from that functional, we will again express the free energy in terms of a fluctuation spectrum $\varepsilon_k$ [see Eq. (9)]. For that, we consider fluctuation modes $\Psi_k = \Psi_0 \exp(i k_x x) \exp(i k_y y) \exp(i k_z d)$, where $|k_z| \leq \pi/d$.

By substitution into Eq. (12), this leads to $\varepsilon_k = \varepsilon + \xi_{ab}(0) k_z \sqrt{(2 \xi_{ab}(0)^2/d^2)(1 - \cos(k_z d))}$. In turn, this $\varepsilon_k$ leads, by using Eq. (10), to:

$$\Delta \sigma_{ab}^{\text{layered}}(\varepsilon) = A_{AL} \left( 1 + B_{LD} \frac{\varepsilon}{\varepsilon} \right)^{-1/2}, \quad (13)$$

where

$$A_{AL} = \frac{\varepsilon^2}{16d} \quad \text{and} \quad B_{LD} = \left( \frac{2 \varepsilon_{c}(0)}{d} \right)^2 \quad (14)$$

are the Aslamazov-Larkin and the Lawrence-Doniach constants.\textsuperscript{3–6,12,14,15,17} Equation (13) reproduces, both in amplitude and in critical exponent, the 2D and 3D results in the limits $\varepsilon \gg B_{LD}$ [i.e., $\xi_c(\varepsilon) \ll d$] and $\varepsilon \ll B_{LD}$ [i.e., $\xi_c(\varepsilon) \gg d$], respectively. Also, it presents an intermediate-dimensionality behaviour in the region around the dimensional crossover reduced-temperature $\varepsilon = B_{LD}$ at which $2 \varepsilon_c(\varepsilon) = d$, which may be calculated as its slope in a log-log representation (i.e., as $\partial \log \Delta \sigma_{ab}^{\text{layered}}(\varepsilon) / \partial \log(\varepsilon)$), takes the value $-3/4$ at $\varepsilon = B_{LD}$, versus the values $-1$ and $-1/2$ of, respectively, the 2D and 3D limits.

For completeness, let us also note that the above GGL expressions for the paraconductivity are not to be compared with our measurements outside the $\varepsilon$-range $0.02 \lesssim \varepsilon \lesssim 0.1$, not only because it is this region where our experimental uncertainties are manageable (see sect. III D), but also because in optimally-doped YBCO the theory is not expected to be accurate both at $\varepsilon \gtrsim 0.1$ and $\varepsilon \lesssim 0.02$: For $\varepsilon > 0.1$, as already indicated in sect. III C, the fluctuations will be dominated by the uncertainty principle limitations to the shrinkage of the superconducting wave function,\textsuperscript{15,17} not taken into account in our present calculations. For $\varepsilon \lesssim 0.02$, the GGL approach fails as the fluctuations are large enough to dominate the physics of the system [instead of perturbative as in Eq. (7)], thus following the full-critical behaviour given by, e.g., the renormalization-group-based 3DXY theory.\textsuperscript{18,19} As already discussed in this journal years ago\textsuperscript{20} and elsewhere,\textsuperscript{1,3,6,12,14,19,21} the $\varepsilon$-boundary between the GGL and the full-critical regions may be estimated by the so-called Levanyuk-Ginzburg reduced-temperature $\varepsilon_{LG}$, defined as the one at which the fluctuation effects on the specific heat become comparable to its mean-field-jump, $c_{\text{jump}}$. For optimally-doped YBCO, the value $\varepsilon_{LG} \approx 0.02$ may be obtained by using the expression valid for layered superconductors,\textsuperscript{21}

$$\varepsilon_{LG} = -\frac{B_{LD}}{4} + \left[ \left( \frac{B_{LD}}{4} \right)^2 + \frac{k_B}{4 \pi c_{\text{jump}}^2(0) d} \right]^{1/2}, \quad (15)$$

and the parameter values\textsuperscript{12,15,21–23} $c_{\text{jump}}(0) = 12 \AA$, $c_{\text{jump}} = 3.5 \times 10^4 \text{J/K}^{-1} \text{m}^{-3}$, $d = 5.85 \AA$ and $B_{LD} = 0.14$. We note that Eq. (15) is in fact not difficult to obtain from our Eq. (9), as shown in Ref. [21]. From a more experimental point of view, there has been observations in optimally-doped YBCO, in the paraconductivity and in other observables, of full-critical 3DXY behaviour at $\varepsilon \lesssim 0.02$,\textsuperscript{12,14,22–24} and also of the uncertainty-principle–dominated behaviour of the fluctuations at $\varepsilon \gtrsim 0.1$.\textsuperscript{15,17} Trying to reproduce those observations falls well beyond the scope of the present article. We indicate in the inset of Fig. 5 the location of these different regions for the superconducting fluctuations above $T_c$ in YBCO.

V. COMPARISON BETWEEN THE EXPERIMENTAL RESULTS AND THE GGL CALCULATIONS FOR $\Delta \sigma_{ab}(\varepsilon)$

The solid lines in Fig. 5 are the results of fitting Eq. (13) to the experiments, in the region $0.02 \lesssim \varepsilon \lesssim 0.1$ and with $B_{LD}$ as the only free parameter.\textsuperscript{25} In these fits the value for $A_{AL}$ was fixed, with a $\pm 30\%$ allowance [corresponding to the uncertainties in the amplitude of the $\Delta \sigma_{ab}(\varepsilon)$ curves discussed in sect. III D], to its theoretical value given by Eq. (14) using the YBCO crystallographic value $d = 5.85 \AA$. Using the same $d$ value, and the $B_{LD}$ resulting from the fit, we obtain $\xi_c(0)$ values between 0.7 and 1.1 $\AA$ (see Fig. 5). This is in good agreement with the result $1.1 \pm 0.2 \AA$ obtained in Refs. [12,15] by measuring the in-plane paraconductivity in high-quality single-crystals and films using a much more complex setup, involving for instance a cryostat with temperature stabilization capability and high-precision lock-in amplifiers. As can be seen, the agreement with the data is excellent. The reduced-temperature corresponding to the dimensional crossover, $\varepsilon = B_{LD}$, is for all the samples inside or near the analyzed $\varepsilon$-region (see Fig. 5), indicating that the proposed experiment probes the crossover between 2D and 3D dimensionality due to the superconducting layered structure.

VI. CONCLUSIONS AND SOME POSSIBLE VARIATIONS

In conclusion, we have presented a simple and affordable experimental setup adequate for probing the effects of thermal fluctuations around the resistive superconducting transition in HTSC. The setup is able to discriminate the critical exponents near $T_c$, as well as the dimensional crossover caused by the layered structure of
these superconductors. This experiment is being successfully used in undergraduate courses of our university. A simple derivation of the equations necessary to understand the phenomenon, appropriate for this student level, has also been presented.

Various modifications to the described program can be made, e.g., to adapt it to shorter or longer courses. For instance, the analysis may be made directly over \( \rho(T) \) instead of \( \rho_{ab}(T) \) [and hence \( \Delta \sigma(\varepsilon) \) instead of \( \Delta \sigma_{ab}(\varepsilon) \)]. In that case, using the fact that the coefficient \( \rho_{01} \) is usually small, one may consider that \( \rho(T) \) and \( \Delta \sigma \) will be roughly proportional to \( \rho_{ab}(T) \) and \( \Delta \sigma_{ab} \), and perform fits to \( \Delta \sigma(\varepsilon) \) with \( A_{\text{AL}} \) as a free parameter, which in this case has to be redefined to also include the unknown polycrystallinity factor \( \alpha \). Note that this freedom in the amplitude of \( \Delta \sigma_{ab}(\varepsilon) \) will not affect the values of the critical exponents (in Fig. 5, it would produce a constant shift in the vertical axis, but not a change of the slopes).

Another possible variation is to analyze the value of \( \tau_0 \) by fitting \( \Delta \sigma_{ab}(\varepsilon) \) with \( A_{\text{AL}} \) and \( B_{\text{LD}} \) as free parameters. As \( \tau_0 \) multiplicatively affects \( \Delta \sigma_{ab} \) (and hence \( A_{\text{AL}} \)), by comparing the obtained \( A_{\text{AL}} \) with the value \( \varepsilon^2/(16\pi d) \) one gets the deviation of the experimental \( \tau_0 \) from the microscopic prediction \( \hbar/(32k_B T_c) \).

It is also feasible to extend the analysis to \( \varepsilon > 0.1 \), and then study the rather sharp disappearance of the superconducting fluctuations for \( \varepsilon \gtrsim 0.5 \), discussed in Refs. [15,17] and references therein. As shown there, the effect may be understood in terms of the limitations imposed by the uncertainty principle to the shrinkage of \( \Psi \) when the coherence length \( \xi(\varepsilon) \) becomes comparable to its value at \( T = 0 \). For performing this analysis a more elaborate subtraction of the background is needed, reducing the large uncertainties at high \( \varepsilon \), shown in Fig. 5. For instance, a two-step procedure similar to the one described in Ref. [15] may be used: first a background is extracted as already explained in subsection III C, and then a second and definitive background is obtained by changing the fitting region to temperatures well above \( \varepsilon \simeq 0.5 \) and by constraining the new fit to be consistent, to within \( \sim 20\% \), with the results for \( \Delta \sigma_{ab}(\varepsilon) \) in the region of \( 0.02 \lesssim \varepsilon \lesssim 0.1 \) obtained with the first background.

VII. ACKNOWLEDGEMENTS

This work has been supported under projects 2006/XA049 and 07TMT007304PR (XUGA-FEDER) and FIS2007-63709 (MEC-FEDER). N. Cotón acknowledges financial support from Spain’s Ministerio de Educación y Ciencia through a FPI grant.

---

Application of Eq. (2) at $T < T_c$ may lead to negative resistivities because, as mentioned in the main text, it is valid only for the normal state where $\rho_{\text{w}}$ and $\alpha$ may be considered constant. Those negative $\rho_{\text{ab}}$ values may be safely replaced with zeros.


The fit of Eq. (13) to the data is, in principle, non-linear. An equivalent linear fit is possible by just rewriting the equation as $\left(\varepsilon \Delta \sigma^\text{layered}_{\text{ab}}\right)^{-2} = A_{\text{XL}}^2 + A_{\text{AL}}^2B_{\text{LD}} \varepsilon^{-1}$. 

References:


